



6.4.4 Zero rows, leading zeros, and leading non-zero entry





Introduction

- In this topic, we will define
 - Zero rows and non-zero rows
 - Leading zeros of a row
 - Leading non-zero entry of a row





Augmented matrices

- Like augmented matrices, the concepts discussed here focus almost entirely on developing an algorithm to solve a system of linear equations
 - They are tools that will be used to find a solution

$$\left(\begin{array}{ccccc|c} 0 & 3.4 & 2.9 & 0 & 5.1 & 9.6 \\ 0 & 0 & 0 & 0 & 0 & 1.1 \\ 1.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.5 & 0 & 5 & 8.3 \end{array} \right)$$





Augmented matrices

- Given an augmented matrix,
every row that is all zeros is said to be a *zero row*
every row that has at least one non-zero entry is said to be
a *non-zero row*

$$\left(\begin{array}{ccccc|c} 0 & 3.4 & 2.9 & 0 & 5.1 & 9.6 \\ 0 & 0 & 0 & 0 & 0 & 1.1 \\ 1.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.5 & 0 & 5 & 8.3 \end{array} \right)$$





Augmented matrices

- Most matrices that engineers work with do not have zero rows by default
 - A zero row simply says $0x_1 + 0x_2 + \dots + 0x_n = 0$

$$\left(\begin{array}{ccccc|c} 0 & 3.4 & 2.9 & 0 & 5.1 & 9.6 \\ 0 & 0 & 0 & 0 & 0 & 1.1 \\ 1.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.5 & 0 & 5 & 8.3 \end{array} \right)$$





Leading zeros

- Given a non-zero row,
the *number of leading zeros* is the number zeros to the left of first non-zero entry in that given row

0	3.4	2.9	0	5.1	9.6	1 leading zero
0	0	0	0	0	1.1	5 leading zeros
1.7	0	0	0	0	0	No or 0 leading zeros
0	0	0	0	0	0	
0	0	5.5	0	5	8.3	2 leading zeros





Leading non-zero entry

- Given a non-zero row,
the first non-zero entry is called the *leading non-zero entry*

$$\left(\begin{array}{ccccc|c} 0 & 3.4 & 2.9 & 0 & 5.1 & 9.6 \\ 0 & 0 & 0 & 0 & 0 & 1.1 \\ 1.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.5 & 0 & 5 & 8.3 \end{array} \right)$$





Leading non-zero entry

- Note that every row is a zero row or has a leading non-zero entry
 - A column is said to have a leading non-zero entry if any entry in that column is a leading non-zero entry of the corresponding row

$$\left(\begin{array}{ccccc|c} 0 & 3.4 & 2.9 & 0 & 5.1 & 9.6 \\ 0 & 0 & 0 & 0 & 0 & 1.1 \\ 1.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.5 & 0 & 5 & 8.3 \end{array} \right)$$





Example

- Looking at another example:
 - Rows 1, 2, 3 and 5 have no leading zeros
 - Row 4 has one leading zero
 - Only Columns 1 and 2 contain leading non-zero entries

$$\left(\begin{array}{ccccc|c} 5.3 & 3.4 & 2.9 & 0 & 5.1 & 9.6 \\ 2.7 & -6.2 & 4.5 & 2.7 & 9.1 & 1.1 \\ 1.7 & 8.4 & 0 & 7.1 & 8.5 & 9.1 \\ 0 & -1.9 & 2.3 & 7.6 & 4.9 & 0 \\ 8.4 & 2.9 & 5.5 & 0 & -5.9 & 8.3 \end{array} \right)$$





Summary

- Following this topic, you now
 - Understand the definition of a zero row and a non-zero row
 - Know how to calculate the number of leading zeros in a row
 - Know how to identify the leading non-zero entry in a non-zero row





Zero rows, leading zeros and leading non-zero entries



References

- [1] https://en.wikipedia.org/wiki/Augmented_matrix





Zero rows, leading zeros and leading non-zero entries



Acknowledgments

None so far.





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Colophon

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